On the Angular Distance of Two Stars in the Pleiades suitable for Determining the Value of a Micrometer Screw. By Professor H. H. Turner, M.A., B.Sc.

1. The following discussion was made in response to a request by Mr. H. F. Newall in 1895 February. He wrote: "I want to determine as accurately as possible the value of my micrometer screw, and have measured two stars in the *Pleiades* about 300″ apart. . . . Can you give me their accurate difference of Right Ascension? To determine this by transits would involve much labour." It is possible that the results of the investigation may be useful to others also.

2. A long series of photographs of the *Pleiades* was taken at this observatory in 1892–93, and some measures on three of these plates have been published (*Monthly Notices*, liv. p. 489). To give a better determination, however, twelve additional plates were measured, as described below.

3. The stars used by Mr. Newall are those numbered 33 and 44 by Elkin.* His numeration will be hereinafter adopted. They are numbered 34 and 45 by Jacoby.†

The second, but not the first, was measured by Bessel in 1840, and called Anon. 24; and again by Pritchard (*Mem. R.A.S.*, vol. xlviii.); but neither by Ambronn, of Göttingen, in 1894. Adopting the precessions and secular variations given by Elkin to bring up both his own and Jacoby's places to 1892·0 (the equinox used for the Oxford photographic measures), we find the differences of Right Ascension and Declination are as follows:—

	R.A.	Decl.
	′ ″	″
Jacoby	+4 50·13	+44·74
Elkin	+4 49·80	+44·75

The Oxford measures already published were made on plates taken on 1892 December 12, 13, and 1893 February 10 (*Monthly Notices*, liv. p. 489), and are expressed in rectangular coordinates on a tangent plane to the celestial sphere at *Alcyone*. It will be convenient to use these coordinates in what follows. The coordinates of the two stars on this plane with axes oriented for 1892·0, and expressed in *réseau* intervals (of 5′ of arc) are, according to Elkin's measures:—

* *Transactions of the Yale University Observatory*, vol. i., part 1, pp. 86 and 87.

† *The Rutherfurd Photographic Measures of the Group of the Pleiades. Observatory of Columbia College.* New York, 1892.

	Star.	ξ	η
	33	$-0{\cdot}8864$	$+2{\cdot}0502$
	44	$-0{\cdot}0034$	$+2{\cdot}1990$
Diff.	(44–33)	$+0{\cdot}8830$	$+0{\cdot}1488$

Jacoby's results, as well as those which follow, may be exhibited as corrections to these differences. The corrections indicated by Jacoby's measures in 1873·0 are—

ξ	η
$+{\cdot}0010$	${\cdot}0000$

The corrections indicated by the three Oxford photographs are—

Plate.	ξ	η
252	$+{\cdot}0006$	$+{\cdot}0013$
259	$-{\cdot}0025$	$+{\cdot}0007$
294	$+{\cdot}0017$	$-{\cdot}0001$
Mean	$-{\cdot}0001$	$+{\cdot}0006$

There seems, therefore, no doubt that the relative proper motion of the two stars is very small, though further measures are desirable.

4. Accordingly on 1895 February 16 I measured the images of eight stars (two images each) on each of eight plates. Two of the stars were the above pair, and the other six were distributed so as to give a good determination of the constants of the plates. The micrometer used was that arranged with a scale in the eye-piece, described in *Monthly Notices*, lv. p. 102. The corrections to the measures due to errors of orientation, &c., of the plate affect the two close stars in question by nearly the same amount; and hence the details of the corrections need not be here given. The method of reduction adopted (by rectangular coordinates) is fully described in *Monthly Notices*, liv. p. 489, and again in lv. p. 102.

5. In Table I., the fifth and ninth columns give the actually measured distances between the stars (parallel to the *réseau* lines, and expressed in *réseau* intervals) for the two images on each plate. Each distance is the difference between two single readings of the eye-piece scale when placed centrally on each star in turn. The third figure is estimated. The next columns give the correction for orientation to the "mean plate" as determined from the four stars numbered by Elkin 19, 41, 43, and 60; and after applying this correction the results, given in columns 7 and 11, are strictly comparable. The differences from the means shown in columns 8 and 12 are not larger than is to be expected, considering that the last digit is estimated, or rather is the difference of two estimations. The errors may be treated as accidental.

TABLE I.

Plate.	Date of Exposure.	Tel. E or W. of pier.	Hour Angle West.	Measured $x_{44}-x_{33}$.	Correction.	Corrected Difference.	Excess over Mean.	Measured $y_{44}-y_{33}$.	Correction.	Corrected Difference.	Excess over Mean.
	1892.		h								
218 (1)	Oct. 17	W.	−3·4	0·890	...	·890	−·001	·151	·000	·151	+·001
(2)			−3·3	·893	...	·893	+·002	·151	·000	·151	+·001
243 (1)	Nov. 29	W.	−0·5	·894	−·001	·893	+·002	·145	+·004	·149	−·001
(2)			−0·3	·895	−·001	·894	+·003	·145	+·003	·148	−·002
244 (1)	Nov. 29	W.	0·0	·891	...	·891	·000	·148	+·003	·151	+·001
(2)			+0·2	·890	...	·890	−·001	·146	+·003	·149	−·001
263 (1)	Dec. 23	W.	−1·3	·892	...	·892	+·001	·148	+·001	·149	−·001
(2)			−1·2	·891	...	·891	·000	·150	+·001	·151	+·001
	1893.										
280 (1)	Jan. 14	W.	+0·1	·889	+·001	·890	−·001	·155	−·006	·149	−·001
(2)			+0·3	·890	+·001	·891	·000	·156	−·006	·150	·000
291 (1)	Feb. 8	E.	+1·5	·889	+·001	·890	−·001	·152	−·004	·148	−·002
(2)			+1·6	·890	+·001	·891	·000	·154	−·004	·150	·000
294 (1)	Feb. 10	W.	+2·1	·890	...	·890	−·001	·149	+·002	·151	+·001
(2)			+2·1	·888	...	·888	−·003	·149	+·002	·151	+·001
306 (1)	Mar. 4	E.	+2·8	·891	...	·891	·000	·149	·000	·149	−·001
(2)			+2·9	·890	...	·890	−·001	·151	·000	·151	+·001
					Mean	·8909			Mean	·1499	

6. Proceeding now to compare the "mean plate" with the theoretical tangent-plane at *Alcyone*, we use the "standard coordinates" ξ and η of the same stars, as given in *Monthly Notices*, liv. p. 494; and with the notation of that paper we find

$$a = e = +\cdot 0078, \; b = -d = \cdot 0000 \text{ sensibly},$$

the values of a and e agreeing well with those found in the above paper, which is a satisfactory indication of the permanence of the scale-value on the plates.

The differences thus become

$$+\cdot 8840 \text{ and } +\cdot 1491$$

as compared with the

$$+\cdot 8830 \text{ and } +\cdot 1488$$

of Elkin, giving corrections to Elkin of

$$+\cdot 0010 \text{ and } +\cdot 0003.$$

7. It may be objected that these measures are made by a comparatively rough method; and this is to some extent true. On the other hand eight different plates were measured and two independent images on each, and the agreement of the different results is fairly satisfactory. Personally, having had some considerable experience of this particular form of micrometer, I am inclined to estimate its capabilities distinctly high; but, as others may not yet agree in this estimate, the following independent investigation was made for the x-coordinates.

Mr. G. F. H. Smith, of New College, Oxford, using the micrometer made by Messrs. Troughton & Simms for the late Professor Pritchard, which is provided with an excellent steel screw ($1^{\text{rev}} = 1^{\text{mm}} = 1'$ approx.), measured the difference of the x coordinates of the two stars 33 and 44 on twelve plates, eight of which were the same as the above. The plates were carefully oriented, so that the direction of measurement was parallel to one set of *réseau* lines, and the corrections for orientation already made use of were thus again available in the case of eight plates. For the others corrections were determined by a similar method. The only new constant required is the value of one revolution of the screw in terms of a *réseau* interval.

8. *Uniformity of Screw.*—The total available length of the screw is about 25 revs. Seven sets of comparisons of different portions of the screw with *réseau* intervals give the following results. It should be remarked that the eye-piece is fixed and the plate bodily moved by the screw, so that a change of focal distance does not affect the measures.

Value of one *réseau* interval in terms of revolutions of screw:

0ʳ — 5ʳ	5ʳ — 10ʳ	10ʳ — 15ʳ	15ʳ — 20ʳ
5ʳ·0039	5·0004	4·9977	4·9966

There is apparently, therefore, a gradual change in the value of 1^{rev} as we proceed along the screw. In the star-measures care was taken always to use the same part of the screw (viz. the portion $5^r - 10^r$) for measuring the distance between the stars 33 and 44; or, in exceptional cases, to apply the corresponding correction.

9. *Value of* 1^{rev} *of Screw.*—This is not assumed to remain constant, since the expansion of the film is probably not the same as that of the screw. At the time of measuring the stars, three or four *réseau* intervals were measured with the screw also. When other parts of the screw than the portion $5^r - 10^r$ were used for either stars or *réseau* intervals, the measures were corrected according to the results of the last paragraph. The mean value of a revolution of the screw, thus found at the time, has been used to reduce the star-measures to *réseau* intervals. The measures are thus expressed in terms of the average *réseau* interval, which is the unit employed in connecting the measures on the plate with Elkin's measures in arc, according to the methods explained above.

10. In the following table, column 5 gives the measure in *réseau* intervals, assuming 5^{rev} of the screw to be equal to one interval. Column 6 gives the correction for runs as in the last paragraph; column 7 the corrections for orientation, &c., found as above, and supplied by Mr. G. F. H. Smith were necessary; column 8 the finally corrected distance; and column 9 the residuals.

Table II.

Plate.	Date of Exp.	Tel. W. or E. of pier.	Hour Angle.	Measure.	Corr. for Runs.	Corr. for Orientation, &c.	Corrected Measure.	Excess over Mean.
			h					
218	Oct. 17	W.	−3·4	·8910	−·0011	−·0074	·8825	−·0009
243	Nov. 29	W.	−0·5	·8909	−·0005	−·0072	·8832	−·0002
244	„ 29	W.	0·0	·8911	−·0002	−·0074	·8835	+ 0001
251	Dec. 12	W.	−2·6	·8912	−·0004	−·0070	·8838	+·0004
254	„ 12	W.	−1·5	·8908	−·0005	−·0066	·8837	+·0003
263	„ 23	W.	−1·3	·8915	−·0008	−·0070	·8837	+·0003
280	Jan. 14	W.	+0·1	·8914	−·0007	−·0058	·8849	+·0015
281	„ 14	W.	+0·7	·8900	−·0005	−·0070	·8825	−·0009
291	Feb. 8	E.	+1·5	·8902	−·0005	−·0062	·8835	+·0001
294	„ 10	W.	+2·1	·8899	− 0003	−·0074	·8822	− 0012
296	„ 14	W.	+0·9	·8900	−·0002	−·0066	·8832	−·0002
306	Mar. 4	E.	+2·8	·8907	−·0004	−·0068	·8835	+·0002
						Mean	·8834	

The mean value thus gives a correction to Elkin of +·0004, as compared with +·0010 from the former measures and −·0001

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from those previously published. The results for the different plates, being each the mean of several readings, naturally accord more closely than before.

11. As one or two of the residuals seem large, Mr. G. F. H. Smith repeated some of his measures; but it seemed clear that the discrepancies were accidental, and nothing is gained by adding these few new measures to the above.

12. Collecting all the results we thus have the following corrections to Elkin's heliometer measures from photographic measures

TABLE III.

Measurer.	Place.	Date of Photos.	Corrections to ξ	Corrections to η
Rutherfurd and Jacoby	New York	1873·0	+·0010	·0000
(Elkin	Yale College	1885·0	·0000	·0000)
Bellamy	Oxford	1893·0	−·0001	+·0006
Turner	,,	,,	+·0010	+·0003
Smith	,,	,,	+·0004	...

Combining the Oxford results with equal weight we should obtain proper motions of −·00003 and +·00001 in ξ and η respectively from the extreme measures. Applying these to the two earlier results we get for the simple mean of New York, Yale, and Oxford, at epoch 1893·0,

$$+\cdot 0002 \text{ and } +\cdot 0001,$$

or in the more familiar unit of seconds of arc

$$\text{in R.A. } +0''\cdot 06 \text{ and in Decl. } +0''\cdot 03,$$

with the annual increments

$$-0''\cdot 009 \text{ and } +0''\cdot 003 \text{ respectively.}$$

13. It would thus appear that, so far as these measures indicate, Elkin's distance between these stars does not at the present time require a correction of so much as $0''\cdot 1$. The following may be given as the definitive value of the distance in arc of a great circle, if it is preferred to use this instead of the difference of R.A.

$$268''\cdot 71 - 0''\cdot 008\ (t - 1893\cdot 0).$$

A General Method for Facilitating the Solution of Kepler's Equation by Mechanical Means. By T. J. J. See, A.M., Ph.D. (Berlin).

The standard works on planetary motion, such as Gauss' *Theoria Motus*, Oppolzer's *Bahnbestimmung*, and Watson's *Theoretical Astronomy*, give methods for solving Kepler's equation which are very satisfactory when the eccentricity of the orbit is small, and also when this element is large, as in the case of most of the periodic comets. When the eccentricity is small, an expansion in series, usually by Lagrange's Theorem, enables us to find the eccentric anomaly with the desired facility. The series frequently employed has the form

$$E_0 = M + e'' \sin M + e''\left(\frac{e}{2}\right) \sin 2M + \ldots .$$

To the approximate value E_0, obtained from a few terms of this series, we apply a correction resulting from the expansion by Taylor's Theorem :

$$E = E_0 + \frac{dE_0}{dM_0} dM_0 + \ldots .$$

The equation of Kepler gives

$$\frac{dM_0}{dE_0} = 1 - e \cos E_0 ;$$

and since

$$dM_0 = M - M_0,$$

we find two terms of the series to be

$$E = E_0 + \frac{M - M_0}{1 - e \cos E_0}.$$

Successive applications of this formula will readily yield the true value of the eccentric anomaly. But when the eccentricity is considerable the expansion in series fails to converge with the desired rapidity. On the other hand, when the orbits differ but little from parabolas, the solution can be readily found by means of special tables, such as those given by Gauss, Watson, and Oppolzer.

It is very remarkable that among the many solutions of Kepler's equation discovered by mathematicians there is not one, so far as I am aware, which has come into general use among astronomers that is applicable to ellipses of all possible eccentricities. The method to which I desire to call attention is a modification of the graphical method given by Klinkerfues,* and originally proposed by Dubois.†

* *Theoretische Astronomie,* p. 17.
† *Astronomische Nachrichten,* No. 1404.

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